Multiscale features of density and frequency spectra from nonlinear gyrokinetics

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Gyrokinetic turbulence simulations covering both electron and ion spatio-temporal scales self-consistently are presented. It is found that for experimentally realistic transport levels at long wavelengths, electron temperature gradient modes may yield substantial or even dominant high-wavenumber contributions to the electron heat flux. It is investigated in which way this behavior is reflected in several experimentally accessible quantities as, for instance, density, electron temperature gradient and frequency spectra. © 2008 American Institute of Physics. [DOI: 10.1063/1.3006086]

I. INTRODUCTION

While it is widely accepted that turbulent fluctuations are the physical origin of the anomalous transport observed in magnetized fusion plasmas, our present understanding of this nonlinear phenomenon is still rather fragmentary. One important open question concerns the role of high-wavenumber (i.e., sub-ion-gyroradius scale) contributions to the electron heat flux. Over the last several years, significant evidence has been collected in both dedicated experimental studies and nonlinear gyrokinetic simulations which suggests that electron temperature gradient (ETG) modes may yield substantial or even dominant high-wavenumber contributions to the electron heat transport under certain conditions, e.g., in plasmas with dominant electron heating, relatively high beta values, substantial equilibrium \( E \times B \) shear, and (internal or edge) transport barriers. This situation triggered a serious effort in the fusion community to extend existing experimental diagnostics into the high wavenumber regime (see, e.g., Refs. 17–20). Unfortunately, the list of observables does not include electron temperature and electrostatic potential fluctuations (or their cross phases). Thus, it is currently not possible to determine electron heat flux spectra directly. Instead, most high-\( k \) diagnostics measure density fluctuation spectra. While the latter may serve as an indicator of the role of turbulence on sub-ion-gyroradius scales, little is known about their connection with electron heat flux spectra. Therefore, the present work is dedicated to an investigation of several experimentally accessible quantities as, for instance, density or frequency spectra by means of nonlinear gyrokinetic simulations covering both electron and ion spatio-temporal scales self-consistently. Our findings are supposed to shed light, in particular, on the link between large ETG-induced transport and recognizable features of the associated density fluctuations.

The present paper is organized as follows: In Sec. II, some background information on the nonlinear gyrokinetic simulations is provided, including the choice of physical and numerical parameters. In Sec. III, wavenumber spectra of the heat and particle fluxes are presented for three basic scenarios, two of which involve large ETG-induced contributions to the electron heat flux. Then, in Secs. IV and V, these three cases are analyzed with respect to the resulting density and frequency spectra. In this context, it will become clear to what extent and how these experimentally observable quantities are related to the transport spectra. The paper closes with some conclusions in Sec. VI.

II. SIMULATION DETAILS

The tool employed for this study is the gyrokinetic turbulence code GENE. Although this code is able to treat magnetic field fluctuations and collisions, these effects are neglected here for simplicity. Moreover, the possibility to extract geometrical information for GENE from magnetohydrodynamic equilibria via the TRACER interface is not used here. Instead, all simulations in this paper are performed in \( \delta - \alpha \) geometry with \( \alpha = 0 \), allowing for a relatively low resolution of 16 grid points in the \( z \) direction parallel to the magnetic field. This, in turn, renders possible the use of up to 768 \( \times \) 384 grid points in the radial \( r \) and binormal \( \theta \) directions which is essential for resolving electron gyroradius scales in simulation domains that are many ion gyroradii wide. Actually, to ensure an adequate coverage of the turbulent structures at long wavelengths, the perpendicular box size is chosen to be \( (L_r,L_\theta) = (64 \rho_i,64 \rho_i) \), where \( \rho_i \) is the ion gyroradius at electron temperature given by \( \rho_i = c_e / \Omega_i \) with ion sound speed \( c_s = \sqrt{T_e/m_i} \) and ion Larmor frequency \( \Omega_i = eB/(m_i c) \). In \( (v||,\mu) \) velocity space, 32 \( \times \) 8 grid points are used.

The physical parameters used in this work are variants of the Cyclone Base Case (CBC) values. All simulations shown below employ a safety factor of \( q = 1.4 \), a magnetic shear of \( \delta = 0.8 \), an inverse aspect ratio of \( e = r / R = 0.18 \), and an equilibrium density/temperature ratio of \( n_i/n_e = 1 \) and \( T_i/T_e = 1 \), respectively. In contrast to the standard CBC scenario, two particle species (electrons and singly charged ions) are kept, both of them fully gyrokinetic. Since the computational effort roughly scales like \( (m_i/m_e)^{3/2} \) (due to the scale separation in \( x, y, \) and \( t \)), we will work with a reduced mass ratio of \( m_i/m_e = 400 \); still, each simulation requires of the order of 100 000 cpu h.

In the following, we will focus primarily on three basic scenarios for which we will discuss transport, density, and...
frequency spectra. They differ with respect to the normalized density and temperature gradients \( R/L_n \) and \( R/L_T \) (\( j = e, i \)) which may deviate from the nominal CBC values. The first one (A) with \( R/L_T = R/L_e = 6.9 \) and \( R/L_n = 2.2 \) reflects a physical situation in which the turbulence is driven (and very strongly) by ion temperature gradient (ITG) modes. Since the resulting value for the ion thermal diffusivity overestimates the experimentally determined one by almost two orders of magnitude,16 we furthermore investigate a case (B) with a reduced low-k drive, characterized by \( R/L_T = 5.5 \), \( R/L_e = 6.9 \), and \( R/L_n = 0.0 \). Here, the high-k fluctuations, driven by ETG modes, contribute significantly to the total electron heat flux, and a scale separation between this transport channel and the remaining ones is clearly visible. Finally, we concentrate on a gradients choice (C) with \( R/L_T = 0.0 \), \( R/L_e = 6.9 \), and \( R/L_n = 0.0 \), in which (only) trapped electron modes (TEMs) and ETG modes are driving the turbulence. This may happen, e.g., in plasmas with dominant electron heating or relatively high beta values. In the next section, we will provide simulation data for the resulting transport spectra in \( k_x \) and \( k_y \) space. After that, density and frequency spectra will be discussed in Secs. IV and V.

III. HEAT AND PARTICLE FLUXES

The electron and ion heat transport spectra in \( k_x \) space for the three cases (A)–(C) have already been presented in Ref. 16. In this context, it was found that for experimentally realistic ion heat (and particle) flux levels in the low-k regime (corresponding to the latter two cases) and in the presence of unstable ETG modes, there tends to be a scale separation between electron and ion thermal transport. In contrast to the latter, the former may exhibit substantial or even dominant high-wavenumber contributions carried by ETG modes and short-wavelength TEMs which are only relevant nonlinearly if ETG modes are unstable. Here, we would like to extend these analyses by also showing particle transport spectra by decomposing the wavenumber contributions according to both \( k_x \) and \( k_y \). For this purpose, contour plots using logarithmically distributed colors are shown in Figs. 1 and 2.

As can be seen in Fig. 1, the ion heat flux spectra are always found to be dominated by wavelengths of the order of many (\( >10 \)) ion gyroradii, in good qualitative agreement with pure large-scale turbulence simulations (see, e.g., Ref. 24). However, the electron heat flux spectra behave differently. Here, the contributions from short-wavelength turbulence increases with decreasing low-k \( (R/L_T) \) drive. Quantitatively, the high-k \( \left( k_y > 1.0 \right) \) fraction of the electron heat transport rises from 10.5% in case (A) to 42.3% in case (B) and finally reaches more than 50% in case (C). In the radial direction, the \( k_y > 1.0 \) fraction in case (A) is 10.6% which is almost identical to the respective \( k_y \) fraction. However, in cases (B) and (C), the high-k contributions amount to values around 30%, therefore implying an anisotropic heat flux spectrum. The physical origin of these high-k anisotropies is most likely the existence of small-scale streamers.6,7,16 This may be the case as long as the low-k drive of the turbulence is of moderate strength. It is probably useful at this point to
recall that case (A) is associated with unrealistically large ion heat fluxes, as has been mentioned before. Therefore, high-\(k\)
transport anisotropies are only to be expected for the other two cases which are, at the same time, to be considered more
relevant. The particle fluxes, presented in Fig. 2, are directed
inwards (describing a particle pinch) in cases (A) and (B), but change sign in case (C) where ITG modes are not excited
anymore. In all three situations, there are no significant
high-\(k\) \( (k\rho_i>1) \) contributions to the particle transport. This
is in line with general expectations based on the fact that the
ions become adiabatic at these scales.

Let us stress again that the three cases (A)-(C) represent
three prototypical examples for two-scale turbulence. Case
(A) corresponds to a system which is driven very (actually,
unrealistically) hard by ITG modes; case (B) describes a situation
in which both ITG and ETG modes contribute to the transport;
and in case (C), ITG modes are absent, leaving all the
turbulent activity to TEMs and ETG modes. These basic
scenarios, together with a few other cases, shall be analyzed
more and more pronounced. Since most of the ETG-induced
transport is located in this wavenumber range and the radial
transport anisotropies are only to be expected for the other
physical parameters or higher resolution. Furthermore, a bulge at high \(k_i\) in the ITG density spectrum

**IV. DENSITY SPECTRA**

Most core turbulence simulations up to date have been
done for situations in which there was only one mode type,
like an ITG mode or a TEM, driving the system. In these
“pure” cases, the density fluctuation spectrum in the binormal
direction, \(S(k_y) = \langle \bar{n}(k_y, \omega) \rangle_{k_z, \omega} \); with \(\langle \cdots \rangle\) denoting averages over quantities listed as indices, usually exhibits a
maximum at \(k_y \rho_i \sim 0.1-0.2\), whereas the radial spectra
\(S(k_x) = \langle \bar{n}(k_x, \omega) \rangle_{k_z, \omega} \) peak at wavenumbers close to zero. At
higher wavenumbers, a power law \(S(k_x) \propto k_x^{-\alpha} \) is typically
seen in both perpendicular directions. Here, the numerically
determined scaling exponents are typically found to be in the
range \(a=3-5\) (see, e.g., Refs. 21, 25, and 26) which is consistent with the experimental findings \(a \sim 3.5 \pm 0.5\) for medium
wavenumbers \(0.3 \leq k_x \rho_i \leq 1.0\) (see, e.g., Ref. 17, and
references therein). For pure ETG turbulence, a similar
behavior has been observed in numerical simulations, where \(\rho_i\)
is replaced by the electron gyroradius \(\rho_e\).\(^{8,27}\)

Examples of highly resolved ITG and temperature gra-
dient driven TEM turbulence simulations with linearly stable
ETG modes are shown in Fig. 3 together with an ETG
turbulence simulation employing a box size restricted to high-\(k\)
modes but retaining nonadiabatic ion dynamics. Here, the
physical parameters are similar to those presented in Sec. II
with the following exceptions. In the ITG simulation the
gradients are \(R/L_T = 6.92\) and \(R/L_n = 0.0\), whereas the
TEM case uses \(R/L_T = 0.0\) and \(R/L_n = 0.0\), and \(T_e/T_i = 3\). The settings for the ETG simulation are the
nominal CBC values, therefore \(R/L_T = R/L_n = 6.92\) and
\(R/L_n = 2.22\). Compared to the publications cited above, the
binormal spectra are slightly flatter, which might be due to the
use of different physical parameters or higher resolution.
Furthermore, a bulge at high \(k_y\) in the ITG density spectrum

![Graph showing spectral behavior](https://example.com/graph.png)

**FIG. 3.** (Color online) Squared electron density fluctuations for pure ITG, TEM, and ETG turbulence cases as a function of radial (a) and binormal (b) wavenumber, each averaged over the remaining directions and time. Since exact characteristics depend strongly on the chosen parameters, these results are only presented to demonstrate that the power law exponent is typically in the range of 2–4, but not necessarily isotropic.

While “pure” turbulence simulations have the great
disadvantage of minimizing the degree of complexity in performing
and analyzing the runs, they usually represent idealized
situations which are, in general, of limited value for direct
comparisons with experimental findings. Thus, a step towards
more realistic simulations involves the study of mix-
tures of two or more different turbulence types as they occur in the
multiscale simulations mentioned before. The corres-
ponding density spectra are presented in Fig. 4. With
increasing ion temperature gradient, a bulge at \(k_y \rho_i \sim 2-5\)
(corresponding to \(k_y \rho_i \sim 0.10-0.25\)) develops and becomes
more and more pronounced. Since most of the ETG-induced
transport is located in this wavenumber range and the radial
spectrum does not show such a distinctive structure, we con-
While the former exhibits a more or less circular shape at high wave numbers or magnetic geometry may also lead to better agreement. Similar arguments apply to the radial direction where Gurchenko and co-workers report a power law transition from a ~2.5 to a ~6.2 at k_yρ_s ~9. Concluding this section, we would like to emphasize that a signature of strong ETG activity is that it tends to flatten the density spectra in the k_yρ_s ~0.1 region. (Note that for a realistic mass ratio of m_i/m_e =1836 or m_i/m_e =3670, this corresponds to k_yρ_s ~4 and k_yρ_s ~6, respectively.) If the long-wavelength dynamics is dominated by ITG modes, the falloff up to that point will still be substantial, however, and presumably no high-k peaks are to be expected. Nevertheless, the ETG-induced contributions to the total electron heat flux can be large since most of it is driven by the positive correlations between fluctuations of the electrostatic potential and the electron temperature, the latter of which tends to decay more slowly than the density fluctuations. The respective spectra of these quantities are shown in Fig. 7, but they can- not be measured in current experiments. Such deviations from isotropy at short wavelengths should be taken into account when comparing numerical with experimental results. This is true, in particular, because in the latter case, the measured k_y spectra often have k_y ~0.17 while in simulations, it is common to average the squared amplitudes over the radial direction. In order to make comparisons easier, we have also evaluated the k_y spectra for k_y=0. The corresponding binormal spectra are shown in Fig. 6. As expected, they differ from those presented in Fig. 3(b) and Fig. 4(b), especially with respect to the power law exponents. One now finds exponents up to a ~5, and if a fit is applied to the range 4 ~ k_yρ_s ~7 in the pure ETG turbulence case, one even arrives at a ~7.4. These values are quite close to the experimental findings17 where a ~3.5 was found at low-k, and a ~6.5 ~7 in the high-k regime. Such characteristics are actually in good qualitative agreement with those of case (C), but the power law exponents do not match. One finds a ~1.9 at 0.15 ~ k_yρ_s ~2 and a ~5.0 at 4 ~ k_yρ_s ~10. The inclusion of Debye shielding effects might reduce the difference since they may steepen the spectrum at high wavenumbers (see, e.g., Ref. 27). A change of plasma parameters or magnetic geometry may also lead to better agreement. Similar arguments apply to the radial direction where Gurchenko and co-workers report a power law transition from a ~2.5 to a ~6.2 at k_yρ_s ~9.28

![FIG. 4. (Color online) Squared electron density fluctuations averaged over the parallel and radial direction and time for turbulence mixtures with R/L_T =6.9 and (A) R/L_T =6.9, (B) R/L_T =5.5, and (C) R/L_T =0.0.](attachment:fig4.png)

![FIG. 5. (Color online) Squared electron density fluctuations as function of k_x and k_y from the multiscale simulations (A) and (C).](attachment:fig5.png)
real frequency at higher wavenumbers suggests that trapped turbulence mixtures, cf. Fig. 3(b). However, nonlinear effects may change the dominant modes, such that one can infer relevant information already from rather inexpensive linear gyrokinetic simulations. This is most pronounced in case A where a steep ion temperature gradient excites ITG modes which can be identified by a positive sign in real frequency and dominate up to a binormal wavenumber of \(k_y \rho_s \approx 0.4\). The abrupt change of sign in real frequency at higher wavenumbers suggests that trapped electron modes drifting in the electron diamagnetic direction take over before they transition into ETG modes. The latter can be clearly identified by their spatio-temporal separation compared to TEM and ITG modes which is given by the square root of ion to electron mass ratio, here 20. However, this plot only shows the most dominant mode for each wavenumber while, in general, several unstable modes may exist at the same wavenumber. In particular, if the strongest subdominant mode is very close to the dominant one, it may alter the nonlinear behavior significantly. Therefore, the \textsc{gene} code has been recently extended to also operate as an eigenvalue solver, enabling the calculation of subdominant modes. The dominant and first subdominant mode for case A are presented in Fig. 9. Due to the real frequency’s sign of the dominant mode the highest growth rate at \(k_y \rho_s \approx 0.5\) can easily be assigned to an ITG mode. At higher wavenumbers, a mode with negative frequency takes over, which may be labelled a TEM-ETG mode since it smoothly transitions from a low-\(k\) TEM to a high-\(k\) ETG mode. Such a transition is not surprising, given that all kinds of microinstabilities may be connected to each other. 

**V. FREQUENCY SPECTRA AND PHASE VELOCITIES**

Another turbulence characteristic which is rather accessible experimentally is a (nonlinear) spectrum of frequencies or phase velocities. Fortunately, as we will see, these quantities are often closely linked to the respective linear quantities, such that one can infer relevant information already from rather inexpensive linear gyrokinetic simulations. However, nonlinear effects may change the dominant mode within a certain \(k\) range with respect to the linear expectations. Such phenomena have to be taken into account when attempting to compare results from experiments and simulations.

According to the growth rate and frequency spectra corresponding to the multiscale simulations (A)–(C), as presented in Fig. 8, we see that different turbulence types are expected to dominate in different wavenumber regimes. This is most pronounced in case (A) where a steep ion temperature gradient excites ITG modes which can be identified by a positive sign in real frequency and dominate up to a binormal wavenumber of \(k_y \rho_s \approx 0.4\). The abrupt change of sign in real frequency at higher wavenumbers suggests that trapped
some surprises. While nonlinear and linear frequencies agree well over a significant region in $k_y$ space in simulation (C), both simulations with unstable ITG modes (A,B) show differences when the ITG mode becomes linearly subdominant at $k_y \rho_s = 0.4$. Here, the real frequency stays predominantly positive up to $k_y \rho_s \approx 1.5$ thus exceeding even the wavenumber where ITG modes are linearly stabilized by almost a factor of 2. At smaller scales, the nonlinear behavior reflects the linear one again to a good approximation. Furthermore, the standard deviations are included in Fig. 10 as error bars. With increasing wavenumber they become larger; thus, it is more difficult to assign a certain frequency to small-scale fluctuations. This may, in part, be due to cross-scale interactions with large-scale turbulence.16 Nevertheless, in all cases shown, the existence of ETG turbulence at high wavenumbers is clearly reflected in the frequency spectra. The respective phase velocities are typically up to a few $c_s \rho_s/R$.

VI. CONCLUSIONS

There is significant experimental and theoretical evidence which suggests that ETG modes may yield substantial or even dominant high-wavenumber contributions to the electron heat transport under certain conditions, e.g., in plasmas with dominant electron heating, relatively high beta values, substantial equilibrium $E \times B$ shear, and transport barriers. Motivated, in particular, by recent experimental advances in the area of high-$k$ density fluctuation diagnostics, we carried out gyrokinetic turbulence simulations with the GENE code, covering both ion and electron spatio-temporal scales self-consistently, and analyzed them with respect to several experimentally accessible quantities like density and frequency spectra.

We found that in contrast to “pure” ITG or TEM turbulence cases, multiscale simulations involving unstable ETG modes tend to exhibit a flat region in the binormal wavenumber spectrum of the density fluctuations at $k_y \rho_s \approx 0.1$. At both longer and shorter wavelengths, power law decays are observed which are more or less in line with respect to earlier,

FIG. 8. (Color online) Linear growth rates (a) and real frequencies (b) at $k_y \rho_s = 0$ vs binormal wavenumber using (A) $R/L_T=R/L_n=6.9$, $R/L_e=2.2$, (B) $R/L_T=5.5$, $R/L_T=6.9$, $R/L_n=0$, and (C) $R/L_T=0$, $R/L_T=6.9$, $R/L_n=0$.

FIG. 9. (Color online) Linear growth rates of dominant and first subdominant mode and dominant real frequency at $k_y \rho_s = 0$ in $c_s/R$ in the small up to medium $k_y$ range for the multiscale simulation (A).
exponent becomes much larger at \( k_y \rho_s \geq 1 \). However, the results do not match quantitatively, most probably because several potentially important physical effects (like collisions, magnetic fluctuations, real geometry, or a finite Debye length) were neglected here for simplicity. We would also like to point out that most experimental measurements were done close to the edge, while our multiscale simulations employed typical core parameters. In addition, increasing the mass ratio to realistic values would lead to a further separation of ion and electron scales and therefore might alter some of the results quantitatively.

Nevertheless, our qualitative findings are expected to remain valid, in particular the fact that high-\( k \) modes may contribute significantly to the electron heat transport although the density spectra might exhibit a rather fast decay. On the other hand, the experimental detection of a flat region in the binormal wavenumber spectrum of the density fluctuations at around \( k_y \rho_s > 0.1 \) would be good evidence for the existence of strong ETG activity. At the same time, our simulations show that measurement of frequencies or phase velocities at short wavelengths can be used to establish the existence of ETG turbulence.

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FIG. 10. (Color online) Dominant real frequency defined as median of Eq. (1) at \( k_y \rho_s \equiv 0 \) for the multiscale simulations (A)–(C) with and without consideration of the nonlinearity. The error bars denote the standard deviations.